

# Absence of fermionic quasi-particles in the superfluid state of the attractive Fermi gas

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We calculate the effect of order parameter fluctuations on the fermionic single-particle excitations in the superfluid state of neutral fermions interacting with short range attractive forces. We show that in dimensions  $D \leq 3$  the singular effective interaction between the fermions mediated by the gapless Bogoliubov-Anderson mode prohibits the existence of well-defined quasi-particles. We explicitly calculate the single-particle spectral function in the BEC regime in  $D = 3$  and show that in this case the quasi-particle residue and the density of states are logarithmically suppressed.

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Recent experimental progress in cooling atomic Fermi gases to ultracold temperatures [1] has revived the interest in the BCS-BEC crossover in the attractive two-component Fermi gas. Although a simple mean-field approximation is sufficient to obtain a qualitatively correct description of the thermodynamics of this crossover [2, 3], for comparison with experiments numerically accurate calculations beyond the mean-field approximation are necessary. Fluctuation corrections to the free energy have been calculated at the level of the ladder approximation [4], which in the functional integral approach is equivalent to the Gaussian approximation in the order parameter fluctuations [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. Close to the unitary point, where the  $s$ -wave scattering length is large, the Gaussian approximation is not sufficient and more sophisticated many-body methods are necessary [16, 17].

While most authors focused on the *thermodynamics* of the BCS-BEC crossover, not much attention has been paid to the *fermionic single-particle excitations* in the BEC regime. Fluctuation corrections to the fermionic self-energy have been briefly considered in the appendix of Ref. [13], but the momentum and frequency dependent single-particle spectral function  $\rho(\mathbf{k}, \omega)$  and the density of states  $\nu(\omega)$  have not been analyzed. In this Letter we shall show that in dimensions  $D \leq 3$  the singular interaction between *neutral* fermions mediated by the gapless Bogoliubov-Anderson (BA) mode leads to the breakdown of the quasi-particle picture. We emphasize that our result is relevant for superfluid gases of cold fermionic atoms, but does not apply to the superconducting state of charged fermions, where the long-range Coulomb interaction pushes the BA mode up to high energies [18].

For our explicit calculations of  $\rho(\mathbf{k}, \omega)$  and  $\nu(\omega)$  we shall focus on the BEC regime in the physically most relevant dimension  $D = 3$  at zero temperature. It turns out that in this case the imaginary part of the self-energy vanishes linearly in the properly shifted frequency variable  $y$  (see below), implying that the frequency dependent real part of the self-energy vanishes as  $y \ln y$ . A similar frequency dependence of the self-energy in the

normal state of strongly correlated electrons has been proposed phenomenologically by Varma *et al.* [19] to explain the unusual normal state properties of the high-temperature superconductors. While a generally accepted theory explaining the microscopic origin of such a marginal Fermi liquid behavior in the *normal* state does not exist, we shall give here a microscopic derivation of similar marginal quasi-particle behavior of the fermionic single-particle excitations in the *superfluid* state of neutral fermions in three dimensions.

To begin with, let us briefly outline the derivation of fluctuation corrections to the fermionic self-energy using functional integration. We consider a system of neutral fermions with dispersion  $\epsilon_{\mathbf{k}} = \mathbf{k}^2/(2m)$  and a short-range attractive two-body interaction  $g_{\mathbf{p}} > 0$  depending on the total two-particle momentum  $\mathbf{p}$ . Writing the fermionic single-particle Green function  $\mathbf{G}$  in the Nambu-Gorkov basis [18] as a functional integral and decoupling the interaction in the spin-singlet particle-particle channel by means of a complex bosonic Hubbard-Stratonovich (HS) field  $\psi$ , we can represent  $\mathbf{G}$  as a functional average,

$$\mathbf{G} = \frac{\int \mathcal{D}[\bar{\psi}, \delta\psi] e^{-S_{\text{eff}}[\bar{\psi}, \delta\psi]} \mathbf{G}_{\psi}}{\int \mathcal{D}[\bar{\psi}, \delta\psi] e^{-S_{\text{eff}}[\bar{\psi}, \delta\psi]}}, \quad (1)$$

where  $\mathbf{G}_{\psi}$  is the fermionic Green function for a fixed configuration of the HS field. Its inverse is given by the following matrix in Nambu-Gorkov and momentum-frequency space [20],

$$[\mathbf{G}_{\psi}^{-1}]_{KK'} = \begin{bmatrix} \delta_{K,K'}(i\omega - \xi_{\mathbf{k}}) & -\psi_{K-K'} \\ -\bar{\psi}_{K'-K} & \delta_{K,K'}(i\omega + \xi_{-\mathbf{k}}) \end{bmatrix}. \quad (2)$$

Here  $\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$  where  $\mu$  is the chemical potential, and the effective action  $S_{\text{eff}}[\bar{\psi}, \delta\psi]$  of the HS field is

$$S_{\text{eff}}[\bar{\psi}, \delta\psi] = \int_P g_{\mathbf{p}}^{-1} \delta\bar{\psi}_P \delta\psi_P + \sum_{n=2}^{\infty} \frac{\text{Tr}[\mathbf{G}_0 \mathbf{V}]^n}{n}, \quad (3)$$

where  $\delta\psi_P = \psi_P - \delta_{P,0}\Delta_0$ , and the mean-field Green function  $\mathbf{G}_0$  is obtained from Eq. (2) by approximating the HS field by its saddle point  $\psi_P \approx \delta_{P,0}\Delta_0$ . The fluctuation matrix  $\mathbf{V}$  is defined by  $\mathbf{V} = \mathbf{G}_0^{-1} - \mathbf{G}_{\psi}^{-1}$ . The saddle

point condition leads to the usual BCS gap equation for  $\Delta_0$  and the quasi-particle dispersion  $E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta_0^2}$ . For convenience we neglect the momentum dependence of  $g_{\mathbf{p}} \approx g_0$  and regularize the resulting ultraviolet divergence [7] in the BCS gap equation by eliminating  $g_0$  in favor of the corresponding two-body  $T$ -matrix in vacuum, which we denote by  $g$ . The relevant dimensionless interaction is then  $\tilde{g} = \nu_0 g$ , where  $\nu_0$  is the density of states at the Fermi energy in the absence of interactions. In  $D = 3$  we have  $\tilde{g} = -2k_F a_s / \pi$ , where  $a_s$  is the  $s$ -wave scattering length in vacuum and  $k_F = mv_F = \sqrt{2m\epsilon_F}$  is the Fermi momentum.

The exact fermionic propagator  $\mathbf{G}$  defined in Eq. (1) can be written as  $\mathbf{G} = [\mathbf{G}_0^{-1} - \Sigma]^{-1}$ , where the self-energy matrix  $\Sigma$  involves a normal component  $\Sigma(K)$  and an anomalous component  $\delta\Delta(K)$  (which can be viewed as a fluctuation correction to  $\Delta_0$ ),

$$[\Sigma]_{KK'} = \delta_{KK'} \begin{bmatrix} \Sigma(K) & \delta\Delta(K) \\ \delta\Delta^*(K) & -\Sigma(-K) \end{bmatrix}. \quad (4)$$

The leading contribution to  $\Sigma$  can be written as  $\Sigma = \langle \mathbf{V}\mathbf{G}_0\mathbf{V} \rangle$ , where  $\langle \dots \rangle$  denotes functional average with the effective action (3) in Gaussian approximation, retaining only quadratic terms in the fluctuations. As emphasized by Castellani *et al.* [21] (see also Ref. [22]), the scaling behavior of the order parameter correlation functions is more transparent if we express the complex field  $\delta\psi_P$  in terms of two real fields [23] by setting  $\delta\psi_P = [\chi_P + i\phi_P]/\sqrt{2}$  and  $\delta\bar{\psi}_P = [\chi_{-P} - i\phi_{-P}]/\sqrt{2}$ . The fields  $\chi_P$  and  $\phi_P$  correspond to longitudinal and transverse fluctuations of the superconducting order parameter, respectively. The effective action (3) in Gaussian approximation is then [20]

$$S_{\text{eff}}[\chi, \phi] \approx \frac{1}{2} \int_P (\chi_P^*, \phi_P^*) \begin{bmatrix} F_P^{\chi\chi} & F_P^{\chi\phi} \\ F_P^{\phi\chi} & F_P^{\phi\phi} \end{bmatrix}^{-1} \begin{pmatrix} \chi_P \\ \phi_P \end{pmatrix}, \quad (5)$$

where for small  $\mathbf{p}$  and  $\bar{\omega}$  the matrix elements of the bosonic propagator are

$$F_P^{\chi\chi} \approx \frac{Dc^2}{\nu_0 v_F^2} \frac{\bar{\omega}^2 + Z_2 c^2 \mathbf{p}^2}{\bar{\omega}^2 + c^2 \mathbf{p}^2}, \quad (6a)$$

$$F_P^{\phi\phi} \approx \frac{Dc^2}{\nu_0 v_F^2} \frac{(2\Delta_0)^2}{\bar{\omega}^2 + c^2 \mathbf{p}^2}, \quad (6b)$$

$$F_P^{\chi\phi} = -F_P^{\phi\chi} \approx \frac{Z_1}{\nu_0} \frac{2\Delta_0 \bar{\omega}}{\bar{\omega}^2 + c^2 \mathbf{p}^2}. \quad (6c)$$

Introducing the dimensionless integrals

$$I_1 = \frac{1}{\nu_0 V} \sum_{\mathbf{k}} \frac{\Delta_0^2}{2E_{\mathbf{k}}^3}, \quad I_2 = \frac{1}{\nu_0 V} \sum_{\mathbf{k}} \frac{\Delta_0 \xi_{\mathbf{k}}}{2E_{\mathbf{k}}^3}, \quad (7)$$

we have  $Z_1 = I_1 I_2 / [I_1^2 + I_2^2]$  and  $Z_2 = [I_1^2 + I_2^2] / I_1^2$ , while the velocity  $c$  of the BA mode can be written as  $Dc^2/v_F^2 = I_1/[I_1^2 + I_2^2]$ . Numerical results for  $Z_1$ ,  $Z_2$  and

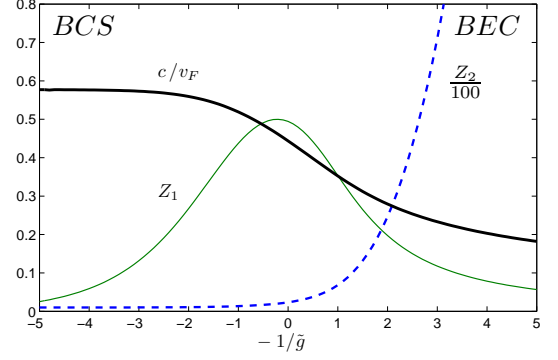


FIG. 1: (Color online) Factors  $Z_1$ ,  $Z_2$ , and  $c/v_F$  defined in the text as a function of  $-1/\tilde{g} = \pi/(2k_F a_s)$  for  $D = 3$ .

$c/v_F$  in  $D = 3$  as a function of the relevant dimensionless coupling  $-\tilde{g}^{-1}$  are shown in Fig. 1. In  $D$  dimensions we obtain in the BCS limit  $c \approx v_F/\sqrt{D}$ , and in the BEC limit  $c = \sqrt{\rho_B U_*/m_B}$ , with the effective interaction  $U_* = \frac{4-D}{D-2}|g|$ . Here  $\rho_B = \rho/2$  and  $m_B = 2m$  are the density and the mass of bosons formed by paired fermions with density  $\rho$  and mass  $m$ . Our result for  $c$  in the BEC limit agrees with the well-known Hartree-Fock result for the velocity of the elementary excitations in the interacting Bose gas provided we identify  $U_*$  with the effective Hartree-Fock potential. Note that only in three dimensions  $U_*$  is given by the zero-energy  $T$ -matrix  $|g|$  of the underlying fermionic system. In the limit  $D \rightarrow 2$  the  $T$ -matrix vanishes as  $g \sim -(D-2)/\nu_0$  (see Ref. [24]), so that in two dimensions  $U_* = 2/\nu_0$  and hence  $c = v_F/\sqrt{2}$  for all values of  $g_0$  [25]. Carrying out the Gaussian average in  $\Sigma = \langle \mathbf{V}\mathbf{G}_0\mathbf{V} \rangle$ , we finally obtain for the fermionic self-energies defined in Eq. (4),

$$\Sigma(K) = -\frac{1}{2} \int_P [F_P^{\phi\phi} + F_P^{\chi\chi} + 2iF_P^{\chi\phi}] B_0(P-K), \quad (8)$$

$$\delta\Delta(K) = -\frac{1}{2} \int_P [F_P^{\phi\phi} - F_P^{\chi\chi}] A_0(P-K), \quad (9)$$

where  $B_0(K) = -[i\omega + \xi_{\mathbf{k}}]/[\omega^2 + E_{\mathbf{k}}^2]$  and  $A_0(K) = -\Delta_0/[\omega^2 + E_{\mathbf{k}}^2]$  are the normal and anomalous components of the mean-field propagator  $\mathbf{G}_0$ .

It turns out that in the BEC limit, where the dimensionless parameter  $\lambda = 4mc^2/\Delta_0 \approx \Delta_0/(2|\mu|)$  is small compared with unity (see Fig. 2), the integrations in Eqs. (8,9) can be explicitly carried out in  $D = 3$  at zero temperature, provided the effective interactions mediated by the bosonic fields are approximated by Eqs. (6a–6c). In this regime the  $s$ -wave bound state energy  $\epsilon_s \approx 2|\mu|$  is the largest energy scale in the problem. Then we may expand the mean-field dispersion as  $E_{\mathbf{k}} \approx E_0 + \mathbf{k}^2/(2m_*)$ , with  $E_0 \approx |\mu| + \Delta_0^2/\epsilon_s = |\mu|(1+2\lambda^2)$  and  $m/m_* \approx 1-2\lambda^2$  for small  $\lambda$ . In fact, the leading non-analytic behavior of the self-energies is entirely due to the infrared singularity in the propagator  $F_P^{\phi\phi}$  of the BA mode in Eq. (6b), so that we may neglect the contributions from  $F_P^{\chi\phi}$  and

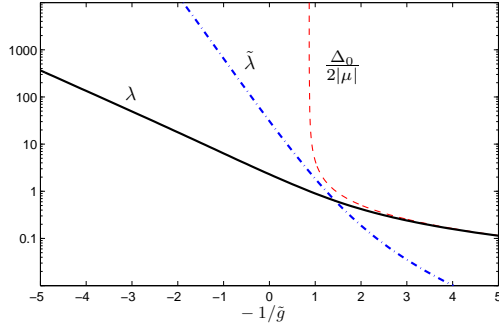


FIG. 2: (Color online) Dimensionless parameter  $\lambda = 4mc^2/\Delta_0$  and  $\tilde{\lambda} = 8\lambda^3/\pi$  in  $D = 3$  as a function of  $-1/\tilde{g}$ . For comparison we also show the ratio  $\Delta_0/(2|\mu|)$ , which approaches  $4mc^2/\Delta_0$  in the BEC limit.

$F_P^{\chi\chi}$  in Eqs. (8,9). Within this approximation, which is accurate for  $|\mathbf{k}|a_s \ll 1$  and  $||\omega| - E_{\mathbf{k}}| \ll |\mu|$ , we obtain

$$\Sigma(\mathbf{k}, i\omega) = -2\Delta_0 [\lambda^2 C(x, z) - (1 + \lambda^2)C(x, z^*)], \quad (10)$$

$$\delta\Delta(\mathbf{k}, i\omega) = 2\Delta_0\lambda [C(x, z) + C(x, z^*)], \quad (11)$$

where  $x = |\mathbf{k}|/k_c$  and  $z = (i\omega - E_{\mathbf{k}})/\omega_c$ , with  $k_c = mc$  and  $\omega_c = mc^2/2$ . The complex function  $C(x, z)$  is

$$\begin{aligned} \pi C(x, z) = & 2 + \ln\left(\frac{M^2}{-z}\right) - \frac{1}{2x} \left\{ \sqrt{(1+x)^2 + z} \right. \\ & \times \ln \left[ \frac{1+x + \sqrt{(1+x)^2 + z}}{1+x - \sqrt{(1+x)^2 + z}} \right] - (x \rightarrow -x) \left. \right\}, \quad (12) \end{aligned}$$

where the ultraviolet cutoff  $M \approx (k_c a_s)^{-1}$  takes into account that in Eqs. (10,11) we have used the long-wavelength approximation (6b) for  $F_P^{\phi\phi}$ . For small  $x$  and  $|z|$  the asymptotic expansion of  $C(x, z)$  is

$$\pi C(x, z) = \ln(M^2/4) + (z/2)\ln(-1/z) + O(z, x^2), \quad (13)$$

so that after analytic continuation  $z \rightarrow y + i0$  we obtain for the imaginary part  $\text{Im} C(x, y + i0) \approx \Theta(y)y/2$  for small  $x$  and  $y$ . The non-analytic term  $z \ln z$  in Eq. (13) can also be derived directly from Eqs. (8,9) by simple power-counting, which reveals for  $D < 3$  an even stronger algebraic singularity. Although our explicit calculation is only valid in the BEC regime, the power counting analysis of Eqs. (8,9) shows that the coupling of the fermions to the gapless BA mode prohibits the existence of well-defined quasi-particles for  $D \leq 3$  in the entire range of the BCS-BEC crossover. This nicely fits to the fact that in the interacting Bose gas the Bogoliubov fixed point is unstable in dimensions  $D \leq 3$ , see Ref. [21].

Given Eqs. (10,11), we may calculate the normal component of the spectral function  $\rho(\mathbf{k}, \omega) = -\pi^{-1} \text{Im} B(\mathbf{k}, \omega + i0)$ , where  $B(K) = B(\mathbf{k}, i\omega)$  is the upper diagonal element of the matrix-propagator  $\mathbf{G}$  defined in Eq. (1). From the derivation of Eqs. (10,11) it is clear that these expressions are only valid for  $|\mathbf{k}| \ll \sqrt{2m|\mu|}$  and  $||\omega| - E_{\mathbf{k}}| \ll |\mu|$ . However, due to the hierarchy of

energy scales  $\omega_c \ll \Delta_0 \ll |\mu|$  in the BEC regime, there are three characteristic regimes where the spectral line-shape exhibits rather distinct behavior:

Regime I:  $\Delta_0 \ll \max\{\epsilon_{\mathbf{k}}, ||\omega| - E_{\mathbf{k}}|\} \ll |\mu|$ . In this regime the spectral function exhibits well-defined quasi-particle peaks at  $\omega = \pm E_{\mathbf{k}}$ , both of which have approximately the same height. The quasi-particle damping is  $\gamma_{\mathbf{k}} = 2\epsilon_s \lambda^3 (|\mathbf{k}|a_s)^4$ , which is small compared with  $E_{\mathbf{k}}$ , so that the fermionic single-particle excitations are well-defined quasi-particles.

Regime II:  $\omega_c \ll \max\{\epsilon_{\mathbf{k}}, ||\omega| - E_{\mathbf{k}}|\} \ll \Delta_0$ . In this intermediate regime the weight of the negative frequency peak is a factor of  $\lambda^2$  smaller than the weight of the positive frequency peak, which agrees with the Hartree-Fock result. There is a considerable asymmetry between the positive and the negative frequency part. The quasi-particle damping in this regime is small and momentum-independent,  $\gamma_{\mathbf{k}} \approx 2\Delta_0 \lambda^4$ .

Regime III:  $\max\{\epsilon_{\mathbf{k}}, ||\omega| - E_{\mathbf{k}}|\} \ll \omega_c$ . Here it is natural to measure momenta and energies in units of the natural scales  $k_c$  and  $\omega_c$  associated with the BA mode. Writing  $\rho(\mathbf{k}, \omega) = \Theta(\omega)\rho_+ + \Theta(-\omega)\rho_-$ , the positive and negative frequency part of the spectral function can be written in scaling form,  $\rho_{\pm}(\mathbf{k}, \omega) = \omega_c^{-1} \tilde{\rho}_{\pm}(x, y)$ , with  $x = |\mathbf{k}|/k_c$  and  $y = (|\omega| - E_{\mathbf{k}})/\omega_c$ . Graphs of the scaling functions  $\tilde{\rho}_{\pm}(x, y)$  are shown in Fig. 3. Note that in regime III, where  $x$  and  $|y|$  are both small compared with unity, there is no spectral weight for  $|\omega| < E_{\mathbf{k}}$ . More generally, for negative  $y$  and  $x < 1 + \sqrt{|y|}$  the spectral function vanishes because  $\text{Im} C(x, y + i0) = 0$  in this regime. Physically, this is due to the kinematic constraint that a quasi-particle with momentum  $\mathbf{k}$  can only spontaneously decay into another quasi-particle and a BA quantum if its velocity  $|\mathbf{k}|/m_*$  exceeds the velocity  $c$  of the BA mode. The scaling functions for  $x, |y| \ll 1$  can be written as  $\tilde{\rho}_+(x, y) = \tilde{\rho}(y)$  and  $\tilde{\rho}_-(x, y) = [\lambda^2 + (y/8)^2] \tilde{\rho}(y)$ , where

$$\tilde{\rho}(y) = \Theta(y)\pi^{-1} Z(y)\gamma(y)/[y^2 + \gamma^2(y)], \quad (14)$$

with  $Z(y) = 1/[1 - \tilde{\lambda} \ln y]$  and the damping function  $\gamma(y) = \pi \tilde{\lambda} y / [1 - \tilde{\lambda} \ln y]$ . The effective coupling parameter  $\tilde{\lambda} = 8\lambda^3/\pi$  is shown in Fig. 2 as a function of  $-\tilde{g}^{-1}$ . For  $\omega \rightarrow E_{\mathbf{k}}$  the damping  $\gamma(y)$  is only logarithmically smaller than the real part of the quasi-particle energy while the quasi-particle residue  $Z(y)$  vanishes logarithmically. Such a behavior resembles the marginal Fermi liquid scenario postulated phenomenologically in Ref. [19].

The spectral function (14) implies a logarithmic reduction of the density of states  $\nu(\omega) = \int \frac{d^3 k}{(2\pi)^3} \rho(\mathbf{k}, \omega)$  for  $\omega \rightarrow E_0 \approx |\mu| + \Delta_0^2/\epsilon_s$ . For positive frequencies and  $\tilde{\lambda} \ln[\omega_c/(\omega - E_0)] \gg 1$  we obtain

$$\frac{\nu(\omega)}{\nu_0} \sim \Theta(\omega - E_0) \sqrt{\frac{\omega - E_0}{\epsilon_F}} \left[ \tilde{\lambda} \ln \left( \frac{\omega_c}{\omega - E_0} \right) \right]^{-1}. \quad (15)$$

The first factor is the usual square-root singularity in  $D = 3$ . For  $\omega \rightarrow -E_0$ , the asymptotic limit of  $\nu(\omega)$  is

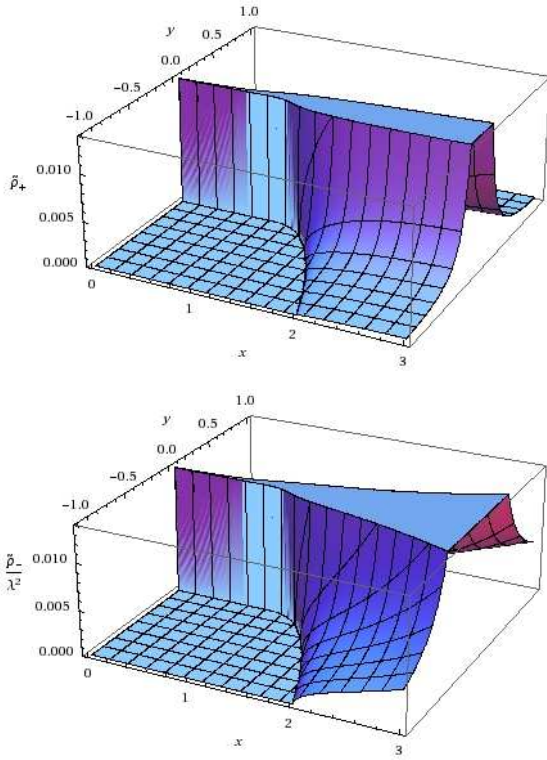


FIG. 3: (Color online) Scaling functions  $\tilde{\rho}_{\pm}(x, y)$  of the single-particle spectral function in the regimes II and III defined in the text. The plots are for  $\lambda = 8\omega_c/\Delta_0 = 0.05$ . In the regime III both  $x$  and  $|y|$  are small compared with unity. For better comparison with  $\tilde{\rho}_+(x, y)$  we have divided  $\tilde{\rho}_-(x, y)$  by  $\lambda^2$ .

a factor of  $\lambda^2$  smaller than for positive frequencies and involves the same logarithmic suppression.

In summary, we have shown that in  $D \leq 3$  the coupling of neutral fermions to the gapless BA mode prohibits the existence of well-defined fermionic quasi-particles in the superfluid state. Although our explicit calculation is only controlled in the BEC limit, we have argued that it remains qualitatively valid for the entire BCS-BEC crossover, because the non-analyticities leading to the breakdown of the quasi-particle picture are due to phase-space restrictions in the infrared regime. In particular, close to the unitary point  $1/\tilde{g} = 0$ , where all energy scales are of the order of  $\epsilon_F$ , we predict a logarithmic suppression of the density of states at the scale  $\epsilon_F$  which should be observable via tunneling experiments or other experimental probes of the density of states in ultracold gases of neutral fermions.

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